CITR
Center for Innovation and Technology Research

CITR Electronic Working Paper Series

Paper No. 2014/3

A Bivariate Integer-Valued Long Memory Model for High Frequency Financial Count Data

A.M.M. Shahiduzzaman Quoreshi

Department of Industrial Economics, Blekinge Institute of Technology, Karlskrona, Sweden
E-mail: shahiduzzaman.quoreshi@bth.se

April 2014

Abstract

We develop a model to account for the long memory property in a bivariate count data framework. We propose a bivariate integer-valued fractional integrated (BINFIMA) model and apply the model to high frequency stock transaction data. The BINFIMA model allows for both positive and negative correlations between the counts. The unconditional and conditional first and second order moments are given. The CLS and FGLS estimators are discussed. The model is capable of capturing the covariance between and within intra-day time series of high frequency transaction data due to macroeconomic news and news related to a specific stock. Empirically, it is found that Ericsson B has mean recursive process while AstraZeneca has long memory property. It is also found that Ericsson B and AstraZenica react in a similar way due to macroeconomic news.

Key Words: Count data, Intra-day, Time series, Estimation, Reaction time, Finance.
JEL Classification: C13, C22, C25, C51, G12, G14.

1 Introduction

This paper introduces a bivariate integer-valued fractionally integrated moving average (BINFIMA) model and applies the model to high frequency stock transaction data. This model is an extension of an INARFIMA model introduced by Quoreshi (2006). This paper focuses on modelling the long memory property of time series of count data in a bivariate setting and on applying the model in a financial setting. The long range dependence or the long memory implies that the present information has a persistent impact on future counts. Note that the long memory property is related to the sampling frequency of a time series. A manifest long memory may be shorter than one hour if observations are recorded every minute, while stretching over decades for annual data. A time series of count data is an integer-valued and non-negative sequence of count observations observed at equidistant instants of time. In the current context series typically have small counts and many zeros. Models for long memory, continuous variable time series are not applicable for integer-valued time series. This is so with respect to both interpretation and inference.

The long memory phenomenon in time series was first considered by Hurst (1951, 1956). In these studies, he explained the long term storage requirements of the Nile River. He showed that the cumulated water flows in a year depends not only on the water flows in recent years but also on water flows in years much prior to the present year. Mandelbrot and van Ness (1968) explain and advance the Hurst's studies by employing fractional Brownian motion. In analogy with Mandelbrot and van Ness (1968), Granger (1980), Granger and Joyeux (1980) and Hosking (1981) develop Autoregressive Fractionally Integrated Moving Average (ARFIMA) models to account for the long memory in time series data. Ding and Granger (1996) point out that a number of other processes can also have the long memory property. An empirical study regarding the usefulness of ARFIMA models is conducted by Bhardwaja and Swanson (2005), who found strong evidence in favor of ARFIMA in absolute, squared and log-squared stock index returns.

This paper introduces a Bivariate Integer-Valued Fractional Integrated Moving Average (BINFIMA) model. The BINFIMA is developed to capture covariance in and between stock transactions time series. Each transaction refers to a trade between a buyer and a seller in a volume of stocks for a given price. A transaction is impounded with information such as volume, price and spread. The trading intensity or the number of transactions for a fixed interval of time and the durations can be seen as inversely related since the more time elapses between successive transactions the fewer trades take place. Easley and O'Hara (1992) shows that a low trading intensity implies no news. Engle (2000) models time according to the autoregressive conditional duration (ACD) model of Engle and Russell (1998) and finds that longer durations are associated with lower price volatilities. Quoreshi (2006a, 2008) introduces Bivariate Integer-Valued Moving Average (BINMA) and Vector Integer-Valued Moving Average Model (VINMA). BINMA model is a special case of VINMA. These models emerge from Integer-Valued Moving Average (INMA) model. The INMA model class has been studied by, e.g. Al-Osh and Alzaid (1988), McKenzie (1988) and Brännäs and Hall (2001). One obvious advantage of the BINMA and VINMA model over extensions of the ACD model is that there is no synchronization problem due to different onsets of durations in count data time series. Hence, the spread of shocks and news is more easily studied in the BINMA or VINMA framework. Like BINMA, the BINFIMA allows for both negative and positive correlation in the count series and the integer-value property of counts is taken into account. The BINFIMA can be considered as a special case of BINMA with infinite lag lengths where the long memory properties are taken into account.

The paper is organized as follows. The BINFIMA models are introduced in Section 2. The conditional and unconditional moment properties of the BINFIMA models are obtained. The estimation procedures, CLS and FGLS for unknown parameters are discussed in Section 3. A detailed description of the empirical data is given in Section 4. The empirical results for the stock series are presented in Section 5 and the concluding comments are included in the final section.

2 BINFIMA and Number of Stock Transactions

Many economic time series, e.g., the number of transactions, the number of car passes during an interval of time, comprise integer-valued count data. It is reasonable to assume that this type of data may also have long memory. However, if employing the previous workhorse, the ARFIMA model, integers cannot be generated. By combining features of the INARMA and ARFIMA models, Quoreshi (2006b) introduced a count data (integer-valued) autoregressive fractionally integrated moving average (INARFIMA) model that takes account of both the integer- valued property of counts and incorporates the long memory property. BINFIMA model is an extension of a special case of INARFIMA model in a bivariate setting.

2.1 The BINFIMA Model

Assume that there are two intra-day series, y_{1t} and y_{2t}, for the number of stock transactions in t = 1, . . . , T time intervals have long memory properties. Assume further that the dependence between y_{1t} and y_{2t} emerges from common underlying factor(s) such as macro-economic news, rumors, etc. Moreover, news related to the y_{1t} series may also have an impact on y_{2t} and vice versa. The covariation within and between the count data variables with long memory properties can be modeled by a BINFIMA model. Like INARFIMA (0,d,0) introduced by Quoreshi (2006b), the model which we call BINFIMA (d_1, d_2) in its simplest form can be defined as follows

$$y_{1t} = u_{1t} + d_{11} \bullet u_{1t-1} + d_{12} \bullet u_{1t-2} + d_{13} \bullet u_{1t-3} \ldots$$

$$y_{2t} = u_{2t} + d_{21} \bullet u_{2t-1} + d_{22} \bullet u_{2t-2} + d_{23} \bullet u_{1t-3} \ldots$$

or

$$y_{1t} = (1 + d_{11} \bullet L + d_{12} \bullet L^2 + d_{13} \bullet L^3 \ldots)u_{1t}$$

$$y_{2t} = (1 + d_{21} \bullet L + d_{22} \bullet L^2 + d_{23} \bullet L^3 \ldots)u_{2t}$$

or

$$y_{it} = (1 + L^{\bullet})^{-d_i} u_{it}. \qquad (1)$$

Note that y_{1t} and y_{2t} have long memory in a sense that the variables have slow decaying autocorrelation functions and the parameters $d_{ij} = \Gamma(j + d_i)/[\Gamma(j + 1)\Gamma(d_i)]$, $i = 1,2$ and $j = 0, 1,2, \ldots$ where $d_0 = 1$. Note that d_{ij} are considered thinning probabilities and hence $d_{ij} \in [0,1]$. It is worth mentioning here that if the parameters are independent and the lag lengths are finites, the model will take the more general form of BINMA (q_1,q_2) Quoreshi (2006a). The macro-economic news are assumed to be captured by $\{u_{it}\}$, $i = 1,2$ and filtered by $\{d_{ij}\}$ through the system. The binomial thinning operator is used to account for the integer-valued property of count data. This operator can be written

$$\alpha \bullet u = \sum_{j}^{u} z_j \qquad (2)$$

with $\{z_j\}_{j=1}^{u}$ an iid sequence of 0-1 random variables, such that $\Pr(z_j = 1) = \alpha = 1\text{-}\Pr(z_j = 0)$. Conditionally on the integer-valued u, $u \bullet \alpha$ is binomially distributed with $E(\alpha \bullet u|u) = \alpha u$ and $V(\alpha \bullet u|u) = \alpha(1-\alpha)u$. Unconditionally it holds that $E(\alpha \bullet u) = \alpha\lambda$ and $V(\lambda \bullet u) = \alpha^2\sigma^2 + \alpha(1-\alpha)\lambda$, where $E(u) = \lambda$ and $V(u) = \sigma^2$. Obviously, $\alpha \bullet u \in [0, u]$.

Assuming independence between and within the thinning operations and $\{u_{it}\}$ an iid sequence with mean λ_i and variance $\sigma_i^2 = v_i\lambda_i$, the unconditional first and second order moments can be given as follows:

$$E(y_{it}) = \lambda_i \left(1 + \sum_{j=1}^{\infty} d_{ij}\right) \qquad (3a)$$

$$V(y_{it}) = \lambda_i \left[\left(v_i + \sum_{j=1}^{\infty} d_{ij}\right) + (v_i - 1)\right]\left(1 + \sum_{i}^{\infty} d_{ij}^2\right) \qquad (3b)$$

$$\gamma_{ik} = \sigma_i^2 \left(d_{ik} + \sum_{j=1}^{\infty} d_{ij} d_{ik+1}\right), \qquad k \geq 1 \qquad (3c)$$

where γ_{ik} denotes the autocovariance function at lag k and $\upsilon_i > 0$. It is obvious from (3) that the mean, variance and autocovariance take only positive values since λ_i , σ^2_i and d_{ij} are all positive and that $\sum_j^{\infty} d_{ij} < \infty$ is required for $\{y_{it}\}$ to be a stationary process. Note also that the variance may be larger than the mean (overdispersion), smaller than the mean (underdispersion), or equal to the mean (equidispersion) depending on whether $\upsilon_i > 1$, $\upsilon_i \in (0, 1)$ or $\upsilon_i = 1$, respectively.

Macro-economic news, rumors, etc. can enhance the intensity of trading in both stocks or lead the intensities in opposite directions. This implies that investors in different stocks may react after the news in similar or different ways. For example, investors may increase their investments in one stock leading to a possible increase in price, while reducing their investments in another stock creating a possible price decrease. Thus, even though the prices of the two stocks move in different directions, the intensities of trading in both stocks may increase. For a fixed time interval [t − 1, t) the macro-economic news are assumed to be captured by u_{it} for stock i. Retaining the previous assumptions and allowing for dependence between u_{1t} and u_{2t} the unconditional covariance function for BINFIMA(d_1,d_2) can be given in the same way like as follows:

$$\gamma_k = \begin{cases} \Lambda\left(d_{1k} + \sum_{j=1}^{q_i-k} d_{1k+j}d_{2j}\right), & 0 \le k \le \min(q_1, q_2) \\ 0, & k > \min(q_1, q_2) > 0 \end{cases} \qquad (4a)$$

$$\gamma_{-k} = \begin{cases} \Lambda\left(d_{2k} + \sum_{j=1}^{q_i-k} d_{2k+j}d_{1j}\right), & 0 \le k \le \min(q_1, q_2) \\ 0, & k > \min(q_1, q_2) > 0 \end{cases} \qquad (4b)$$

where $q_i \rightarrow \infty$ and $\gamma_k = Cov\ (y_{1t}, y_{2t-k})$, $\gamma_{-k} = Cov\ (y_{1t-k}, y_{2t})$ and $Cov\ (u_{1t}, u_{2t}) = \Lambda = \varphi - \lambda_1\lambda_2$ where $\varphi = E(u_{1t}u_{2t})$. Note here that in empirical studies q_i may be very large but always finite. In its empirical implication, we assume the coefficients approaches zero as lag lengthen approaches infinite. There is no cross-lag dependence among u_{it} and the covariances Cov(u_{1t}, u_{2t}) are assumed constant over time. Note also that the sign of the covariance function in (4a−b) depends on the relative sizes of φ and $\lambda_1\lambda_2$. Quoreshi (2006a) gives the conditional mean and variance for a BINMA (q_1, q_2) model. The conditional mean, variance and covariance for the BINFIMA (d_1,d_2) are in an analogous way

$$E(y_{it}|Y_{it-1}) = E_{i|t-1} = \lambda_i\left(1 + \sum_{j=1}^{\infty} d_{ij}\, u_{it-j}\right) \qquad (5a)$$

$$V(y_{it}|Y_{it-1}) = V_{i|t-1} = \lambda_i v_i + \sum_{j=1}^{\infty} d_{ij}(1 - d_{ij})u_{it-j} \qquad (5b)$$

$$\gamma_{k|t-1} = E[(y_{1t} - E_{1|t-1})(y_{2t-k} - E_{2t-k-1})|Y_{1t-1}, Y_{2t-k-1} \qquad (5c)$$
$$= \begin{cases} \Lambda, & k = 0 \\ 0, & \text{otherwise} \end{cases}$$

where Y_{it-1} is the information set available at time *t-1*. The conditional mean and variance vary with u_{it-j} while the conditional covariance does not. Since the conditional variance varies with u_{it-j}, there is a conditional heteroskedasticity property of moving average type that Brännäs and Hall called MACH(q). The effect of u_{it-j} on the mean is greater than on the variance. Note also that like the unconditional variance the conditional variance could be overdispersed, underdispersed or equidispersed depending on whether $v_i > 1 + \sum_j^{\infty} d_{ij}/\lambda_i, v_i \in (1 + \sum_j^{\infty} d_{ij}/\lambda_i)$ or $v_i = 1 + \sum_j^{\infty} d_{ij}/\lambda_i,$ respectively.

2.2 Extension of the BINFIMA Model

A number of other processes may have long memory (Ding and Granger, 1996). Like INRFIMA (0,δ,0) proposed by Quoreshi (2006), the BINFIMA model can be extended in the following way

$$y_{it} = \theta(d_i) \bullet (1 + L^{\bullet})^{-d_i} u_{it}. \qquad (6)$$

Where $d_{ij} = \Gamma(j + d_i)/[\Gamma(j + 1)\Gamma(d_i)]$, $i = 1{,}2$ and $j = 0, 1{,}2, \ldots$ where $d_0 = 1, \theta(d_i) = b_i{}^{-d_i}$ where b_i are some constants and the property $\varphi_1 \bullet (\varphi_2 \bullet v) \stackrel{\text{def}}{=} (\varphi_1\varphi_2) \bullet v$ is employed. The coefficients in this expression are considered thinning probabilities and hence we require $\theta(d_i), d_i \in [0,1]$. We denote the model in (6) BINFIMA (δ_1,δ_2). The conditional mean, variance and covariance of the model are

$$E(y_{it}|Y_{it-1}) = E_{i|t-1} = \theta(d_i)\lambda_i\left(1 + \sum_{j=1}^{\infty} d_{ij}\, u_{it-j}\right) \qquad (7a)$$

$$V(y_{it}|Y_{it-1}) = V_{i|t-1} = \theta^2(d_i)\sigma_i^2 + (1 - \theta^2(d_i))\lambda_i + \sum_{j=1}^{\infty} \theta(d_i)d_{ij}\big(1 - \theta(d_i)d_{ij}\big)u_{it-j} \qquad (7b)$$

$$\gamma_{k|t-1} = E[(y_{1t} - E_{1|t-1})(y_{2t-k} - E_{2t-k-1})|Y_{1t-1}, Y_{2t-k-1} \qquad (7c)$$
$$= \begin{cases} \Lambda, & k = 0 \\ 0, & \text{otherwise.} \end{cases}$$

Note that when $\theta^2(d_i) = 1$, BINFIMA (δ_1,δ_2) collapses to BINFIMA (d_1,d_2).

3. Estimation

Since we do not assume a full density function the maximum likelihood estimator is not considered. Conditional least square (CLS), feasible generalized least square (FGLS) and generalized methods of moments (GMM) are first hand candidates for the estimation. In the previous studies, it turns out that FGLS is the best estimator among the three in terms of eliminating serial correlation (Quoreshi, 2006). CLS comes in the second position. Hence, we will consider CLS and FGLS for estimation. The estimators, CLS and FGLS, for BINFIMA (δ_1,δ_2) have the following residual in common

$$e_{i1t} = y_{it} - E_{i|t-1} = y_{it} - \theta(d_i)\lambda_i\left(1 + \sum_{j=1}^{\infty} d_{ij}\, u_{it-j}\right). \qquad (8)$$

These moment conditions correspond to the normal equations of the CLS estimator that focuses on the unknown parameters of the conditional mean function. Alternatively and equivalently the properties E(e_{i1t}) = 0 and E($e_{i1t}e_{i1t-j}$) = 0, $j \geq 1$ could be used. The criterion function $S_i = \sum_{i=m+1}^{T} e_{1t}^2$is minimized with respect to unknown parameters, i.e. $\psi_i = \big(\lambda_i, \theta(d_i)$ and $d_{ij}\,\big)$. Using a finite maximum lag m in (8) instead of infinite lags may have biasing effects. Due to omitted variables, i.e. $u_{it-m-1}, \ldots, u_{it-\infty}$, we may have expect a positive biasing effect on the parameters $\lambda_i, \theta(d_i)$ and d_{ij} (Brännäs and Quoreshi, 2010). Note that the moment conditions for an BINFIMA (d_1,d_2) can be obtained by setting $\theta(d_i) = 1$.

The parameters estimated with CLS are considered a first step of the FGLS estimators. The conditional variance and the covariance prediction errors

$$e_{i2t} = (y_{it} - E_{i|t-1})^2 - V_{i|t-1} \quad (9)$$
$$e_{3t} = e_{i1t}e_{i2t} - \gamma_{0|t-1} \quad (10)$$

are used. For FGLS, $S_{i2} = \sum_{t=s_i}^{T} e_{i2t}^2$ and $S_3 = \sum_{t=s_i}^{T} e_{3t}^2$, where $s_i = q_i + 1$ The moment estimators for σ_i^2 and φ can be written on the following forms

$$\hat{\sigma}_i^2 = (T - s_i)^{-1} \sum_{t=s_i}^{T} \left[e_{i1t}^2 - \sum_{j=0}^{q_i} \theta(d_i)d_{ij}\big(1 - \theta(d_i)d_{ij}\big)u_{it-j} \right]$$

$$\hat{\varphi} = (T - s)^{-1} \sum_{t=s+1}^{T} e_{i1t}e_{i2t} - \hat{\lambda}_1\hat{\lambda}_2$$

where s=max(q_1,q_2)-1 and T is the length of time series. Finally, FGLS estimator minimizes the criterion

$$S_{FGLS} = \sum\nolimits_{t=s_j}^{T} \frac{\hat{V}_{2|t-1}e_{11t}^2 + \hat{V}_{1|t-1}e_{21t}^2 - 2\hat{\gamma}_{0|t-1}e_{11t}^2e_{21t}^2}{\hat{D}_t} \quad (15)$$

is minimized with respect to ψ_i. In (15) $\hat{V}_{i|t-1}$, $\hat{\gamma}_{0|t-1}$ and $\hat{D}_t = \hat{V}_{1|t-1}\hat{V}_{2|t-1} - \hat{\gamma}_{0|t-1}$ are taken as given. This gives the FGLS estimates of the parameter vector $\psi = (\psi_1', \psi_2')'$ of the bivariate conditional mean function. The covariance matrix estimator is

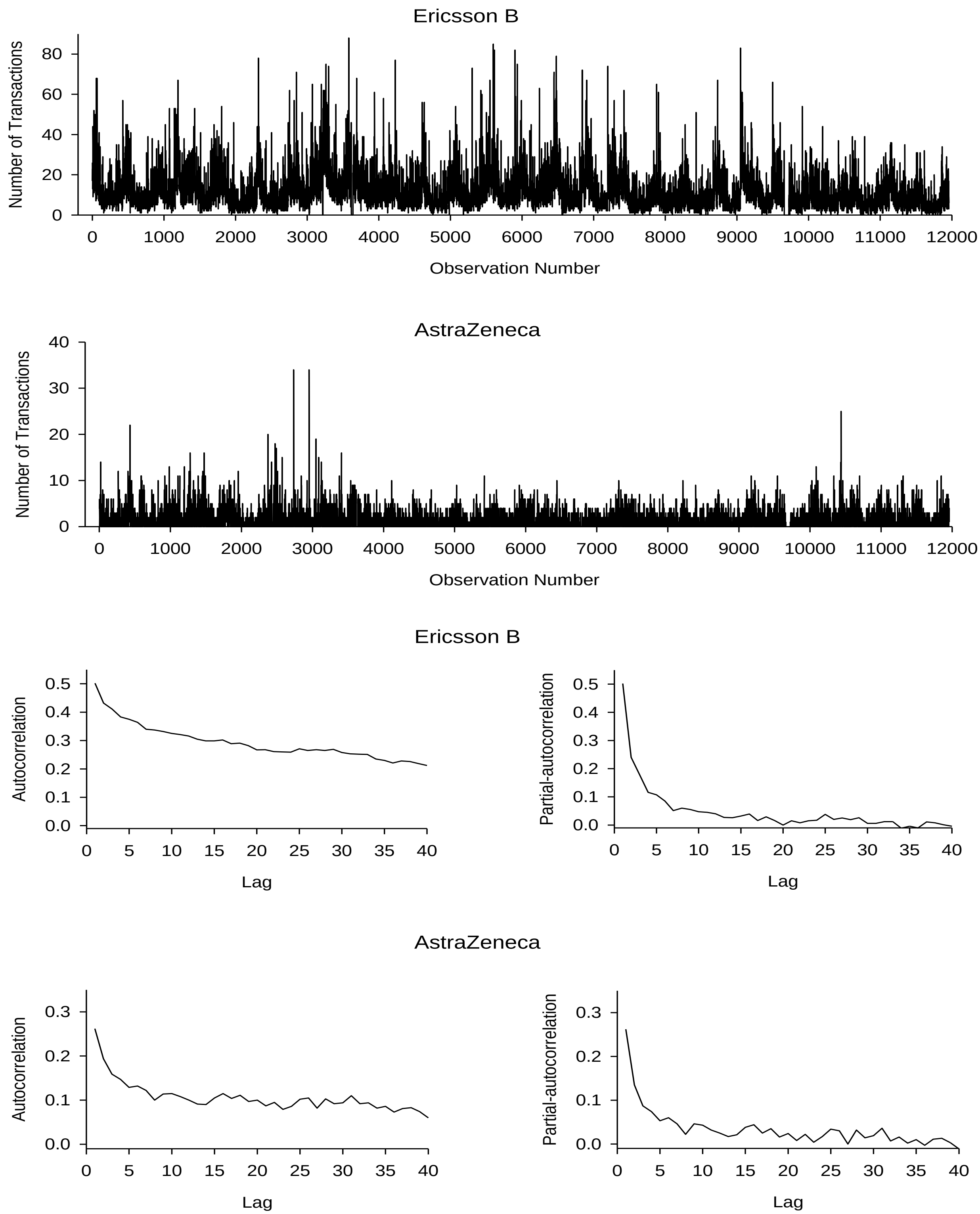
$$Cov(\hat{\psi}_{FGLS}) = \left(\sum_{t=s+1}^{T} \frac{\partial e_t}{\partial \psi'} \hat{\Omega}^{-1} \frac{\partial e_t}{\partial \psi} \right)^{-1}$$

Where $e_t = (e_{11t}e_{21t})'$and $\hat{\Omega}$ is the covariance matrix for the residual series FGLS estimation.

4. Data and Descriptives

The tick-by-tick data for Ericsson B and AstraZeneca have been downloaded from the Ecovision system and are later filtered by the author. The stocks are frequently traded and have the highest turnovers at the Stockholmsbörsen. The two stock series are collected for the period November 5-December 12, 2002. Due to a technical problem in downloading data there are no data for November 12 in the time series and the first captured minutes of December 5 are 0959 and 1037, respectively. Since we are interested in capturing the number of ordinary transactions, we have deleted all trading before 0935 (trading opens at 0930) and after 1714 (order book closes at 1720). The transactions in the first few minutes are subject to a different trading mechanism while there is practically no trading after 1714. The data are aggregated into one minute intervals of time. For high frequency data, researchers usually use one, two, five or ten minute intervals of time and the choice is rather arbitrary. There are altogether 11960 observations for both the Ericsson B and AstraZeneca series. The series together with their autocorrelation and partial-autocorrelation functions are exhibited in Figure 1. There are frequent zero frequencies in both series, specially in the AstraZeneca series and hence the application of count data modeling is called for. The counts in both series fluctuate around their means which is an indication of mean reverting processes. The autocorrelation functions for both series suggest fractional integration.

Figure 1: The time series of Ericsson B and AstraZeneca and their autocorrelation and partial-autocorrelation functions.



5. Empirical Results

CLS and FGLS estimators are employed and AIC criterion are used to determine the lag length. Between BINFIMA (d_1,d_2) and BINFIMA (δ_1,δ_2), the later turns out better in terms of eliminating serial correlation, although the differences are very small. The parameters estimated with CLS and FGLS are almost the same (Table 1). BINMA turns out best among the three in terms of eliminating serial correlation. We found evidence for fractional integration property for both Ericsson B and AstraZeneca series. The series for Ericsson B has mean reversion property but is not covariance stationary since the confidence interval for δ includes 0.5. The series for AstraZenica has mean reversion property and is covariance stationary. The correlation between Ericsson B and AstraZeneca is positive which implies that both series move in the same direction when macroeconomic news or rumors break out. For BINFIMA (δ_1,δ_2) we need to estimate 6 parameters, while the number of estimated parameters for the BINMA (70, 50) are 122. The time to run a BINFIMA model took about 20 minutes while the corresponding time for a BINMA was about 14 hours. Hence, BINFIMA is more parsimonious in terms of parameters and estimating time than those of BINMA. Note also that a few parameters for Ericsson B estimated with BINMA turned out negative which is a violation for that the parameters are interpreted as probabilities and hence take values between zero and one (see Table 3). Obviously, we could find positive parameters by employing, for example, better start-up values or forcing parameters to be turned out positive. The mean and median reaction times for Ericsson B estimated with BINFIMA are 22 minutes respective 13 minutes, while the corresponding numbers for BINMA are 20 respective 12 minutes. But the differences in median are rather large (see Table 2). The parameters estimated with BINMA model is given in Table 3 in the appendix.

Table 1: Results for BINFIMA (δ_1,δ_2) model for Ericsson B and AstraZeneca estimated with CLS and FGLS.

	Ericsson B				AstraZeneca			
	CLS		FGLS		CLS		FGLS	
	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
$\hat{\delta}$	0.486	0.024	0.486	0.023	0.357	0.049	0.358	0.048
$\hat{b}$	1.239	0.112	1.238	0.110	0.498	0.106	0.485	0.107
$\hat{\theta}(\delta)$	0.548	0.103	0.548	0.103	0.837	0.149	0.841	0.129
$\hat{\lambda}$	2.401	0.120	2.379	0.119	0,597	0.072	0.593	0.077
$\hat{\sigma}^2$	42.73		42.74		1.98		1.98	
$\hat{\varphi}$	2.734		2.731					
$\hat{\gamma}_{k\|t-1}$	1.301		1.319					
LB_{100}	193.5		192.8		209.6		208.7	
LB_{200}	272.8		272.0		330.7		329.2	

Table 2: FGLS estimation results for BINFIMA and BINMA models for Ericsson B and AstraZenica.

	BINFIMA		BINMA	
	Ericsson B	AstraZeneca	Ericsson B	AstraZeneca
RT_m	21.67	13.29	19.89	11.90
RT_{me}	16.29	15.31	9,26	5.98
LB_{100}	192.8	208.7	172.8	198.7
$\hat{\rho}_{0\|t-1}$	0.309		0,339	

6. Concluding Remarks

This paper focuses on modeling the long memory property in a bivariate count data framework. The proposed BINFIMA model emerges from INARFIMA model introduced by Quoreshi (2006b). CLS and FGLS estimators are discussed. The model can also be seen as a special case of BINMA model. The BINFIMA model is more parsimonious than BINMA both in terms of parameters and estimating time. In its empirical application, we found evidence for fractional integration property for both Ericsson B and AstraZeneca series. The series for Ericsson B has mean reversion property but is not covariance stationary since the confidence interval for δ includes 0.5. The series for AstraZenica has mean reversion property and is covariance stationary. The correlation between Ericsson B and AstraZeneca is positive which implies that both series move in the same direction when macroeconomic news or rumors break out.

Appendix

Table 3: FGLS estimation results for BINMA (70, 50) model Ericsson B and AstraZeneca.

Ericsson B						AstraZeneca					
Lag	$\hat{\beta}_j$	s.e*10	Lag	$\hat{\beta}_j$	s.e*10	Lag	$\hat{\beta}_j$	s.e*10	Lag	$\hat{\beta}_j$	s.e*10
1	0.285	0.067	36	0.025	0.07	1	0.016	0.020	36	0.008	0.012
2	0.200	0.073	37	0.031	0.07	2	0.020	0.018	37	0.007	0.013
3	0.192	0.076	38	0.030	0.07	3	0.012	0.016	38	0.004	0.011
4	0.156	0.081	39	0.022	0.08	4	0.010	0.020	39	0.004	0.013
5	0.162	0.083	40	0.008	0.07	5	0.013	0.016	40	0.004	0.013
6	0.161	0.089	41	0.016	0.08	6	0.014	0.013	41	0.007	0.010
7	0.132	0.099	42	0.007	0.07	7	0.013	0.014	42	0.003	0.013
8	0.133	0.104	43	0.009	0.07	8	0.013	0.013	43	0.008	0.011
9	0.134	0.105	44	0.018	0.07	9	0.012	0.012	44	0.007	0.012
10	0.118	0.108	45	0.017	0.07	10	0.006	0.014	45	0.005	0.015
11	0.123	0.104	46	0.010	0.07	11	0.009	0.011	46	0.002	0.013
12	0.122	0.106	47	0.018	0.07	12	0.008	0.013	47	0.007	0.014
13	0.114	0.107	48	0.005	0.06	13	0.011	0.011	48	0.008	0.017
14	0.100	0.097	49	0.022	0.07	14	0.010	0.012	49	0.006	0.015
15	0.104	0.099	50	-0.003	0.07	15	0.010	0.012	50	0.003	
16	0.117	0.097	51	-0.011	0.07	16	0.010	0.011			
17	0.101	0.098	52	-0.019	0.07	17	0.011	0.012			
18	0.111	0.099	53	-0.024	0.07	18	0.014	0.017			
19	0.105	0.095	54	-0.012	0.07	19	0.011	0.015			
20	0.081	0.102	55	-0.019	0.07	20	0.007	0.020			
21	0.083	0.086	56	-0.032	0.07	21	0.013	0.019			
22	0.072	0.085	57	-0.050	0.07	22	0.011	0.026			
23	0.071	0.086	58	-0.030	0.07	23	0.012	0.014			
24	0.066	0.080	59	-0.023	0.07	24	0.013	0.018			
25	0.084	0.077	60	-0.025	0.06	25	0.012	0.014			
26	0.070	0.075	61	-0.028	0.07	26	0.011	0.012			
27	0.076	0.075	62	-0.020	0.07	27	0.007	0.013			
28	0.072	0.067	63	-0.026	0.07	28	0.007	0.013			
29	0.082	0.068	64	-0.023	0.06	29	0.011	0.014			
30	0.072	0.072	65	-0.018	0.06	30	0.009	0.019			
31	0.067	0.068	66	-0.035	0.06	31	0.011	0.009			
32	0.070	0.074	67	-0.030	0.06	32	0.008	0.013			
33	0.070	0.073	68	-0.009	0.06	33	0.008	0.011			
34	0.053	0.069	69	-0.019	0.06	34	0.009	0.012			
35	0.046	0.072	70	-0.019	0.04	35	0.009	0.014			
$\hat{\lambda}$	2.45					$\hat{\lambda}$	0.63				
$\hat{\sigma}^2$	42.80					$\hat{\sigma}^2$	2.35				

References

Al-Osh, M.A. and Alzaid, A.A. (1987). First Order Integer-Valued Autoregressive INAR(1)) Process. Journal of Time Series Analysis 8, 261-275.

Bhardwaj, G. and Swanson, N.R. (2005). An Empirical Investigation of the Usefulness of ARFIMA Models for Predicting Macroeconomic and Financial Time Series. Journal of Econometrics 131, 539-578.

Brännäs, K. and Hall, A. (2001). Estimation in Integer-Valued Moving Average Models. Applied Stochastic Models in Business and Industry 17, 277-291.

Brännäs, K. and Quoreshi, A.M.M.S. (2010). Integer-Valued Moving Average Modelling of the Number of Transactions in Stocks. Applied Financial Economics 20, 129-1440.

Ding, Z. and Granger, C.W.J. (1996). Varieties of Long Memory Models. Journal of Econometrics 73, 61-77.

Easley, D. and O'Hara, M. (1992). Time and the Process of Security Price Adjustment. Journal of Finance 47, 905-927.

Engle, R.F. (2000). The Econometrics of Ultra-High Frequency Data. Econometrica 68, 1-22.

Engle, R.F. and Russell, J.R. (1998). Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data. Econometrica 66, 1127-1162.

Ding, Z. and Granger, C.W.J. (1996). Varieties of Long Memory Models. Journal of Econometrics 73, 61-77.

Granger, C.W.J. (1980). Long Memory Relationships and the Aggregation of Dynamic Models. Journal of Econometrics 14, 227-238.

Granger, C.W.J. and Joyeux, R. (1980). An Introduction to Long-Memory Time Series Models and Fractional Differencing. Journal of Time Series Analysis 1, 15-29.

Hosking, J.R.M. (1981). Fractional Differencing. Biometrika, 68, 165-176.

Hurst, H.E. (1951). Long-Term Storage Capacity of Reservoirs. Transactions of the American Society of Civil Engineers 116, 770-808.

Hurst, H.E. (1956). Methods of Using Long-Term Storage in Reservoirs. Proceedings of the Institute of Civil Engineers 1, 519-543.

McKenzie, E. (1986). Autoregressive Moving-Average Processes with Negative Binomial and Geometric Marginal Distributions. Advances in Applied Probability 18, 679-705.

Mandelbrot, B.B. and Van Ness, J.W. (1968). Fractional Brownian Motions, Fractional Noises and Applications. SIAM Review 10, 422-437.

Quoreshi, A.M.M.S. (2006a). Bivariate Time Series Modelling of Financial Count Data. Communications in Statistics—Theory and Methods 35, Issue 7. Forthcoming.

Quoreshi, A.M.M.S. (2006b). A Long Memory, Count Data, Time Series Model for Financial Application. Umeå Economic Studies 673.

Quoreshi, A.M.M.S (2008). A vector integer-valued moving average model for high frequency financial count data. Economics Letters 101 (2008) 258–261